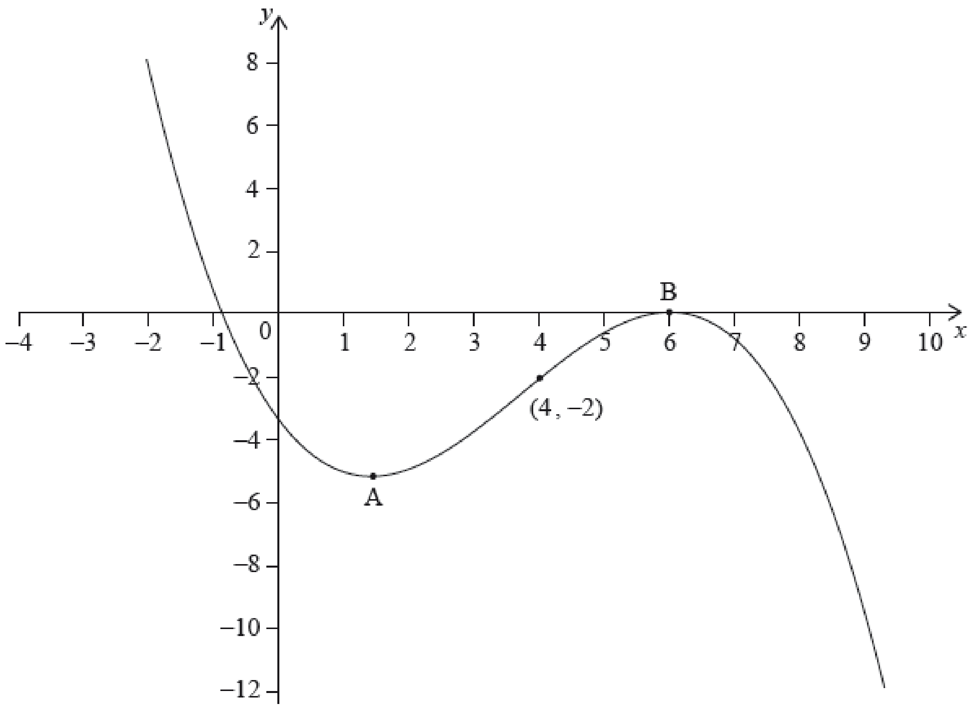
SL Paper 1

The following diagram shows the graph of f', the derivative of f.



The graph of f' has a local minimum at A, a local maximum at B and passes through $(4, \ -2)$.

The point $\mathrm{P}(4, \ 3)$ lies on the graph of the function, f.

a.i. Write down the gradient of the curve of f at P. [1]

a.ii. Find the equation of the normal to the curve of f at P. [3]

b. Determine the concavity of the graph of f when $4 < x < 5$ **and** justify your answer. [2]

The values of the functions f and g and their derivatives for $x = 1$ and $x = 8$ are shown in the following table.

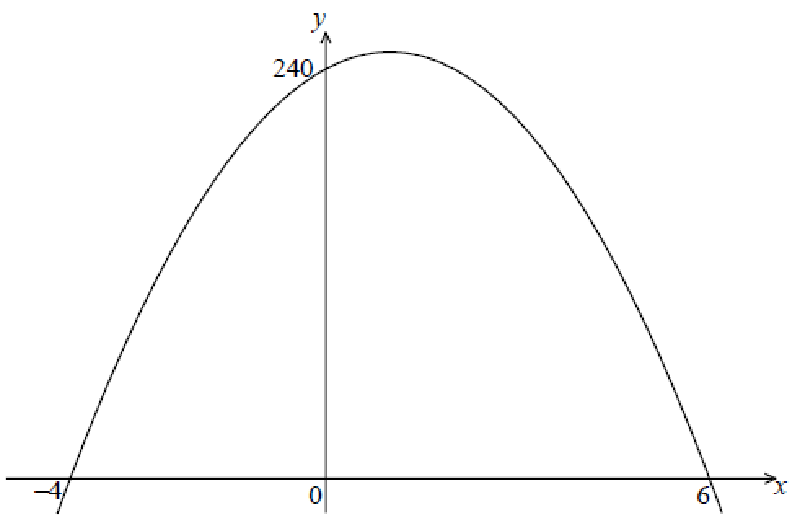
x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	4	9	−3
8	4	−3	2	5

Let $h(x) = f(x)g(x)$.

a. Find $h(1)$. [2]

b. Find $h'(8)$. [3]

The following diagram shows part of the graph of a quadratic function f.



The x-intercepts are at $(-4, 0)$ and $(6, 0)$, and the y-intercept is at $(0, 240)$.

a. Write down $f(x)$ in the form $f(x) = -10(x-p)(x-q)$. [2]

b. Find another expression for $f(x)$ in the form $f(x) = -10(x-h)^2 + k$. [4]

c. Show that $f(x)$ can also be written in the form $f(x) = 240 + 20x - 10x^2$. [2]

d(i) And (ii). A particle moves along a straight line so that its velocity, v ms^{-1}, at time t seconds is given by $v = 240 + 20t - 10t^2$, for $0 \le t \le 6$. [7]

(i) Find the value of t when the speed of the particle is greatest.

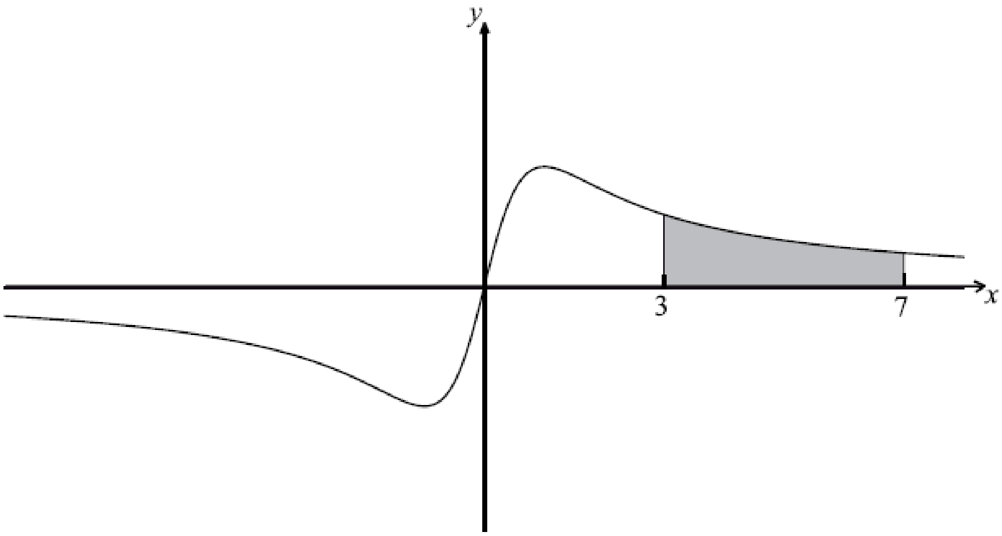
(ii) Find the acceleration of the particle when its speed is zero.

Let $g(x) = 2x \sin x$.

a. Find $g'(x)$. [4]

b. Find the gradient of the graph of g at $x = \pi$. [3]

Let $f(x) = \frac{ax}{x^2+1}$, $-8 \le x \le 8$, $a \in \mathbb{R}$. The graph of f is shown below.



The region between $x = 3$ and $x = 7$ is shaded.

a. Show that $f(-x) = -f(x)$. [2]

b. Given that $f''(x) = \frac{2ax(x^2-3)}{(x^2+1)^3}$, find the coordinates of all points of inflexion. [7]

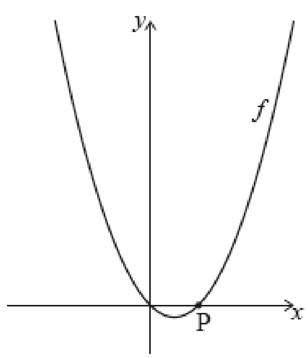
c. It is given that $\int f(x)\mathrm{d}x = \frac{a}{2}\ln(x^2+1) + C$. [7]

(i) Find the area of the shaded region, giving your answer in the form $p \ln q$.

(ii) Find the value of $\int_4^8 2f(x-1)\mathrm{d}x$.

Let $f(x) = x^2 - x$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f.

diagram not to scale



The graph of f crosses the x-axis at the origin and at the point $\mathrm{P}(1,\ 0)$.

The line *L* is the normal to the graph of *f* at P.

The line L intersects the graph of f at another point Q, as shown in the following diagram.

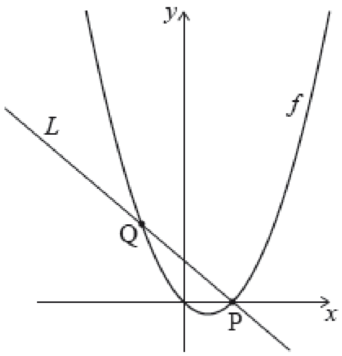


diagram not to scale

a. Show that $f'(1) = 1$. [3]

b. Find the equation of L in the form $y = ax + b$. [3]

c. Find the x-coordinate of Q. [4]

d. Find the area of the region enclosed by the graph of f and the line L. [6]

Let $f(x) = \frac{\cos x}{\sin x}$, for $\sin x \neq 0$.

In the following table, $f'\left(\frac{\pi}{2}\right) = p$ and $f''\left(\frac{\pi}{2}\right) = q$. The table also gives approximate values of $f'(x)$ and $f''(x)$ near $x = \frac{\pi}{2}$.

x	$\frac{\pi}{2} - 0.1$	$\frac{\pi}{2}$	$\frac{\pi}{2} + 0.1$
$f'(x)$	-1.01	p	-1.01
$f''(x)$	0.203	q	-0.203

a. Use the quotient rule to show that $f'(x) = \frac{-1}{\sin^2 x}$. [5]

b. Find $f''(x)$. [3]

c. Find the value of p and of q. [3]

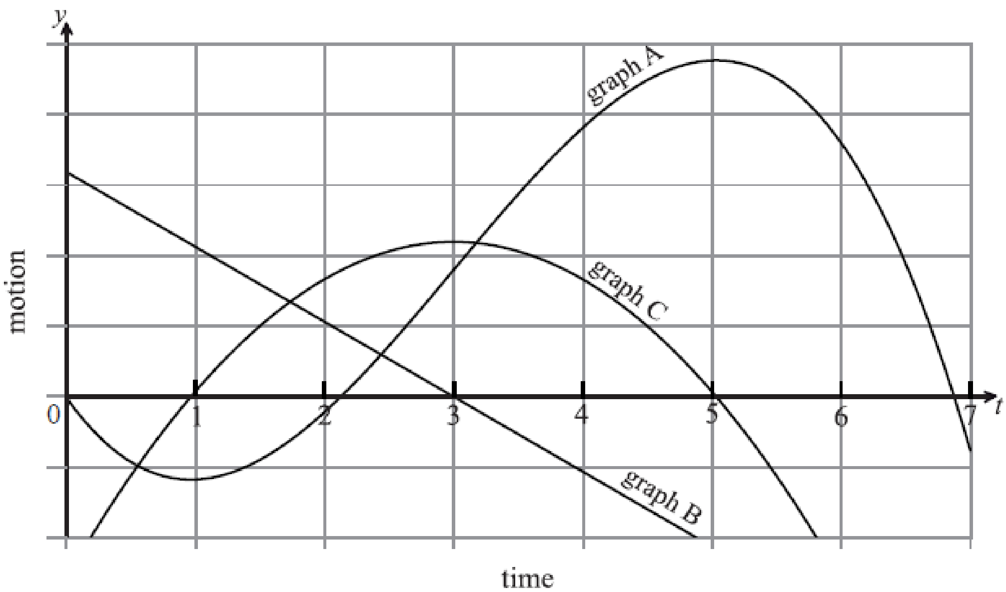
d. Use information from the table to explain why there is a point of inflexion on the graph of f where $x = \frac{\pi}{2}$. [2]

Consider $f(x) = x^2 + \frac{p}{x}$, $x \neq 0$, where p is a constant.

a. Find $f'(x)$. [2]

b. There is a minimum value of $f(x)$ when $x = -2$. Find the value of p . [4]

The following diagram shows the graphs of the **displacement**, **velocity** and **acceleration** of a moving object as functions of time, t.

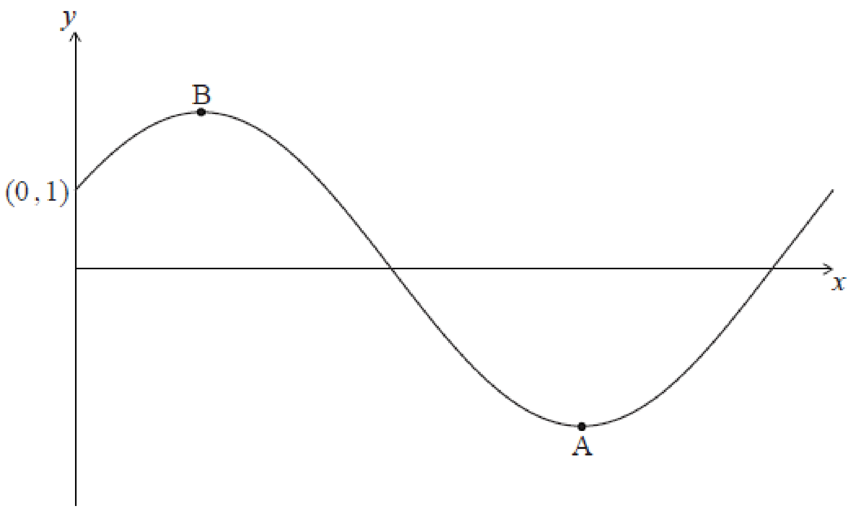


a. Complete the following table by noting which graph A, B or C corresponds to each function. [4]

Function	Graph
displacement	
acceleration	

b. Write down the value of t when the velocity is greatest. [2]

Let $f(x) = \cos x + \sqrt{3} \sin x$, $0 \le x \le 2\pi$. The following diagram shows the graph of f .



The y-intercept is at (0, 1) , there is a minimum point at A (p, q) and a maximum point at B.

a. Find $f'(x)$. [2]

b(i) and (ii). Hence [10]

(i) show that $q = -2$;

(ii) verify that A is a minimum point.

c. Find the maximum value of $f(x)$. [3]

d. The function $f(x)$ can be written in the form $r\cos(x - a)$. [2]

Write down the value of r and of a .

Consider a function f. The line L_1 with equation $y = 3x + 1$ is a tangent to the graph of f when $x = 2$

Let $g(x) = f\left(x^2 + 1\right)$ and P be the point on the graph of g where $x = 1$.

a.i. Write down $f'(2)$. [2]

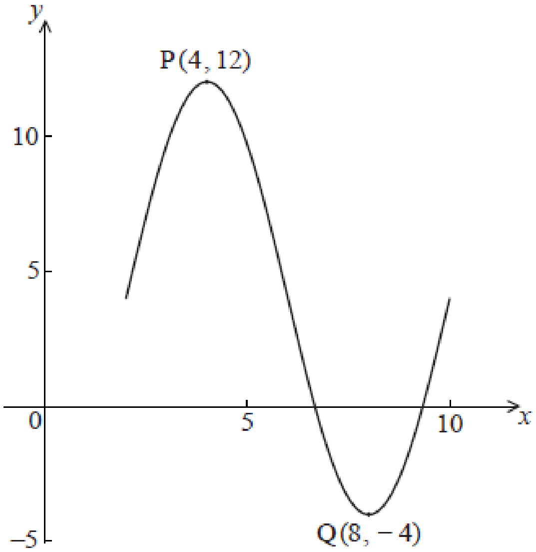
a.ii. Find $f(2)$. [2]

b. Show that the graph of g has a gradient of 6 at P. [5]

c. Let L_2 be the tangent to the graph of g at P. L_1 intersects L_2 at the point Q. [7]

Find the y-coordinate of Q.

The following diagram shows the graph of $f(x) = a\sin(b(x - c)) + d$, for $2 \le x \le 10$.



There is a maximum point at P(4, 12) and a minimum point at Q(8, −4) .

a(i), (ii) and (iii). Use the graph to write down the value of [3]

(i) a ;

(ii) c ;

(iii) d .

b. Show that $b = \frac{\pi}{4}$. [2]

c. Find $f'(x)$. [3]

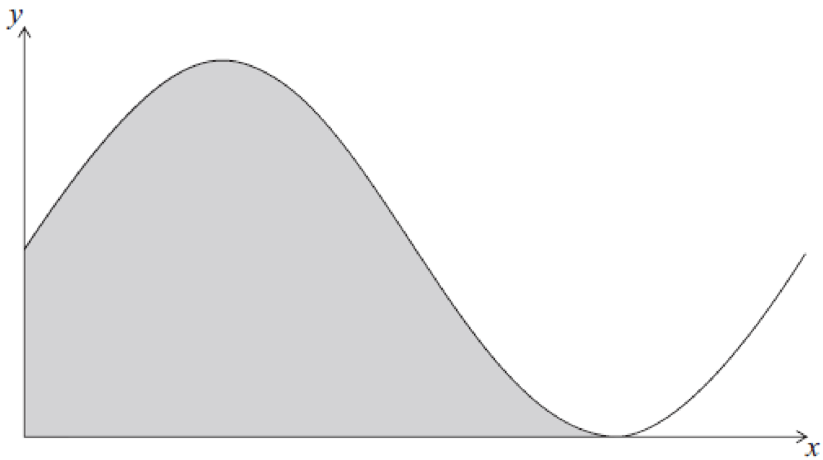
d. At a point R, the gradient is -2π . Find the x-coordinate of R. [6]

Given that $f(x) = \frac{1}{x}$, answer the following.

a. Find the first four derivatives of $f(x)$. [4]

b. Write an expression for $f^{(n)}(x)$ in terms of x and n . [3]

Let $f(x) = 6 + 6\sin x$. Part of the graph of f is shown below.



The shaded region is enclosed by the curve of f, the x-axis, and the y-axis.

a(i) and (ii). Solve for $0 \le x < 2\pi$ [5]

(i) $6 + 6\sin x = 6$;

(ii) $6 + 6\sin x = 0$.

b. Write down the exact value of the x-intercept of f, for $0 \le x < 2\pi$. [1]

c. The area of the shaded region is k . Find the value of k , giving your answer in terms of π . [6]

d. Let $g(x) = 6 + 6\sin\left(x - \frac{\pi}{2}\right)$. The graph of f is transformed to the graph of g. [2]

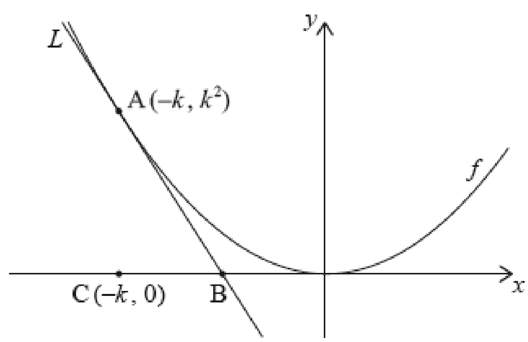
Give a full geometric description of this transformation.

e. Let $g(x) = 6 + 6\sin\left(x - \frac{\pi}{2}\right)$. The graph of f is transformed to the graph of g. [3]

Given that $\int_p^{p+\frac{3\pi}{2}} g(x)\mathrm{d}x = k$ and $0 \le p < 2\pi$, write down the two values of p.

Let $f(x) = x^2$. The following diagram shows part of the graph of f.

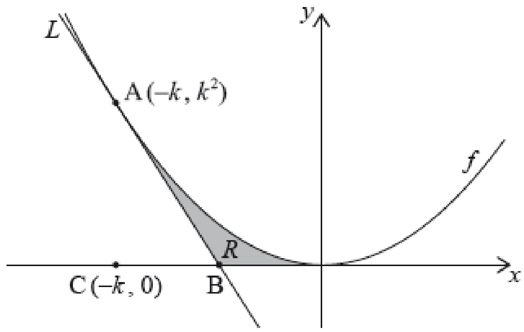
diagram not to scale



The line L is the tangent to the graph of f at the point $\mathrm{A}(-k,\ k^2)$, and intersects the x-axis at point B. The point C is $(-k,\ 0)$.

The region R is enclosed by L, the graph of f, and the x-axis. This is shown in the following diagram.

diagram not to scale



a.i. Write down $f'(x)$. [1]

a.ii. Find the gradient of L. [2]

b. Show that the x-coordinate of B is $-\frac{k}{2}$. [5]

c. Find the area of triangle ABC, giving your answer in terms of k. [2]

d. Given that the area of triangle ABC is p times the area of R, find the value of p. [7]

Let $f(x) = \sqrt{4x+5}$, for $x \geqslant -1.25$.

Consider another function g. Let R be a point on the graph of g. The x-coordinate of R is 1. The equation of the tangent to the graph at R is $y = 3x + 6$.

a. Find $f'(1)$. [4]

b. Write down $g'(1)$. [2]

c. Find $g(1)$. [2]

d. Let $h(x) = f(x) \times g(x)$. Find the equation of the tangent to the graph of h at the point where $x = 1$. [7]

Let $f(x) = \cos x$.

Let $g(x) = x^k$, where $k \in \mathbb{Z}^+$.

Let $k = 21$ and $h(x) = \left(f^{(19)}(x) \times g^{(19)}(x)\right)$.

a. (i) Find the first four derivatives of $f(x)$. [4]

(ii) Find $f^{(19)}(x)$.

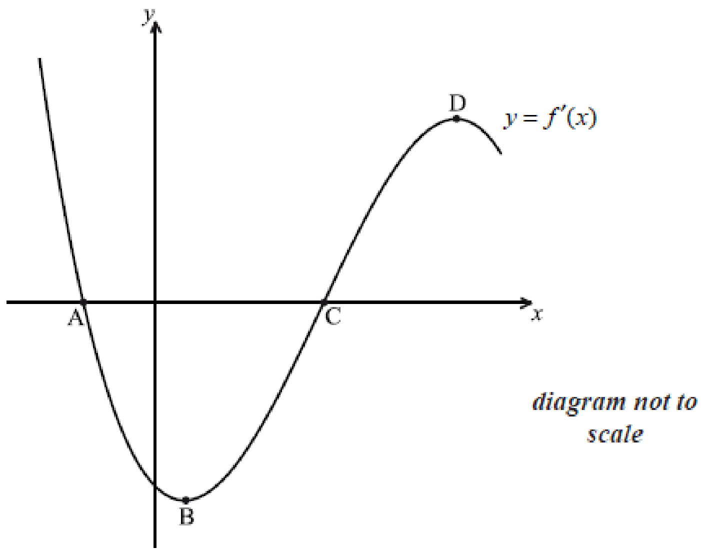
b. (i) Find the first three derivatives of $g(x)$. [5]

(ii) Given that $g^{(19)}(x) = \frac{k!}{(k-p)!}(x^{k-19})$, find p.

c. (i) Find $h'(x)$. [7]

(ii) Hence, show that $h'(\pi) = \frac{-21!}{2}\pi^2$.

The diagram shows part of the graph of $y = f'(x)$. The x-intercepts are at points A and C. There is a minimum at B, and a maximum at D.



a(i) and (ii) (i) Write down the value of $f'(x)$ at C. [3]

(ii) **Hence**, show that C corresponds to a minimum on the graph of f, i.e. it has the same x-coordinate.

b. Which of the points A, B, D corresponds to a maximum on the graph of f? [1]

c. Show that B corresponds to a point of inflexion on the graph of f. [3]

Let $f(x) = e^{6x}$.

a. Write down $f'(x)$. [1]

b(i) and (ii) The tangent to the graph of f at the point $P(0, b)$ has gradient m . [4]

(i) Show that $m = 6$.

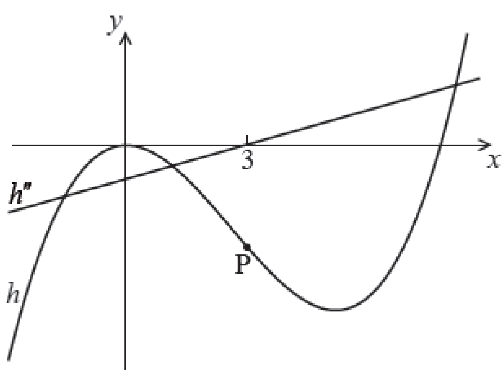
(ii) Find b .

c. Hence, write down the equation of this tangent. [1]

Consider the functions $f(x)$, $g(x)$ and $h(x)$. The following table gives some values associated with these functions.

x	2	3
$f(x)$	2	3
$g(x)$	−14	−18
$f'(x)$	1	1
$g'(x)$	−5	−3
$h''(x)$	−6	0

The following diagram shows parts of the graphs of h and h'' .



There is a point of inflexion on the graph of h at P, when $x = 3$.

Given that $h(x) = f(x) \times g(x)$,

a. Write down the value of $g(3)$, of $f'(3)$, and of $h''(2)$. [3]

b. Explain why P is a point of inflexion. [2]

c. find the y-coordinate of P. [2]

d. find the equation of the normal to the graph of h at P. [7]

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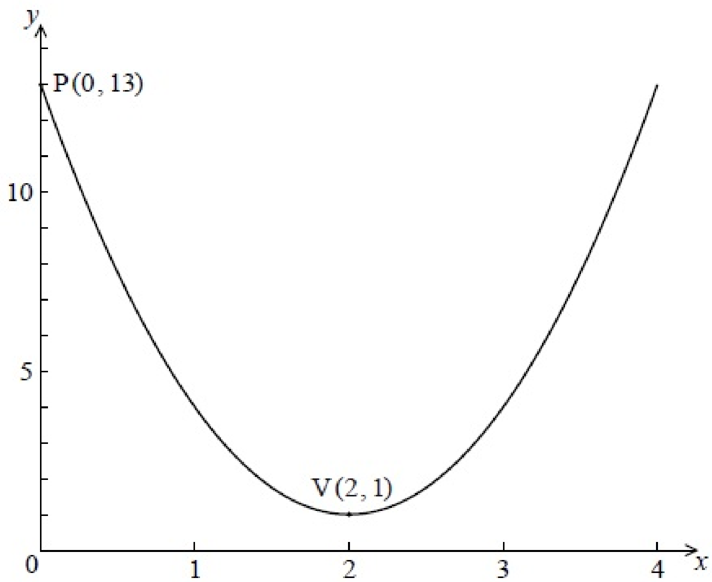
a. Find $\int \frac{1}{2x+3} \mathrm{d}x$. [2]

b. Given that $\int_0^3 \frac{1}{2x+3} \mathrm{d}x = \ln \sqrt{P}$, find the value of P. [4]

The following diagram shows the graph of a quadratic function f, for $0 \leq x \leq 4$.



The graph passes through the point P(0, 13) , and its vertex is the point V(2, 1) .

a(i) and (ii). The function can be written in the form $f(x) = a(x - h)^2 + k$. [4]

(i) Write down the value of h and of k .

(ii) Show that $a = 3$.

b. Find $f(x)$, giving your answer in the form $Ax^2 + Bx + C$. [3]

c. Calculate the area enclosed by the graph of f, the x-axis, and the lines $x = 2$ and $x = 4$. [8]

In this question s represents displacement in metres and t represents time in seconds.

The velocity v m s^{-1} of a moving body is given by $v = 40 - at$ where a is a non-zero constant.

Trains approaching a station start to slow down when they pass a point P. As a train slows down, its velocity is given by $v = 40 - at$, where $t = 0$ at P. The station is 500 m from P.

a. (i) If $s = 100$ when $t = 0$, find an expression for s in terms of a and t. [6]

(ii) If $s = 0$ when $t = 0$, write down an expression for s in terms of a and t.

b. A train M slows down so that it comes to a stop at the station. [6]

(i) Find the time it takes train M to come to a stop, giving your answer in terms of a.

(ii) Hence show that $a = \frac{8}{5}$.

c. For a different train N, the value of a is 4. [5]

Show that this train will stop **before** it reaches the station.

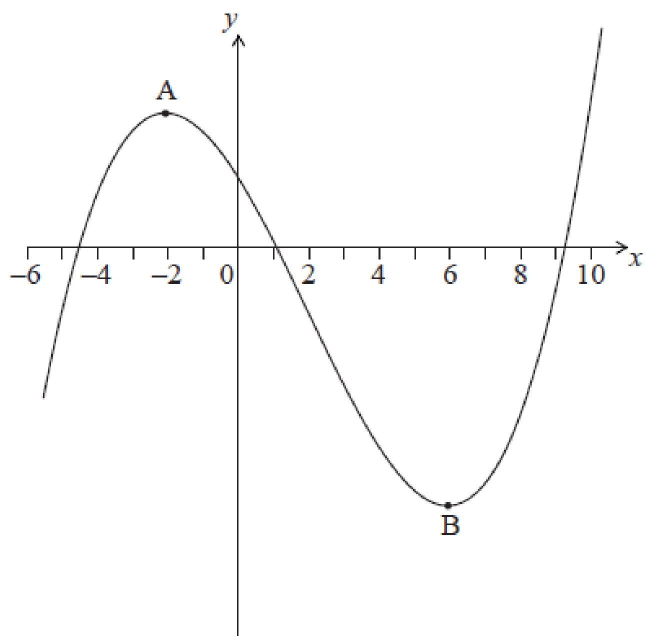
a. Find $\int x\mathrm{e}^{x^2-1}\mathrm{d}x$. [4]

b. Find $f(x)$, given that $f'(x) = x\mathrm{e}^{x^2-1}$ and $f(-1) = 3$. [3]

Let $f(x) = kx^4$. The point $\mathrm{P}(1,\, k)$ lies on the curve of f . At P, the normal to the curve is parallel to $y = -\frac{1}{8}x$. Find the value of k.

The graph of the function $y = f(x)$ passes through the point $\left(\frac{3}{2}, 4\right)$. The gradient function of f is given as $f'(x) = \sin(2x - 3)$. Find $f(x)$.

The following diagram shows part of the graph of $y = f(x)$.

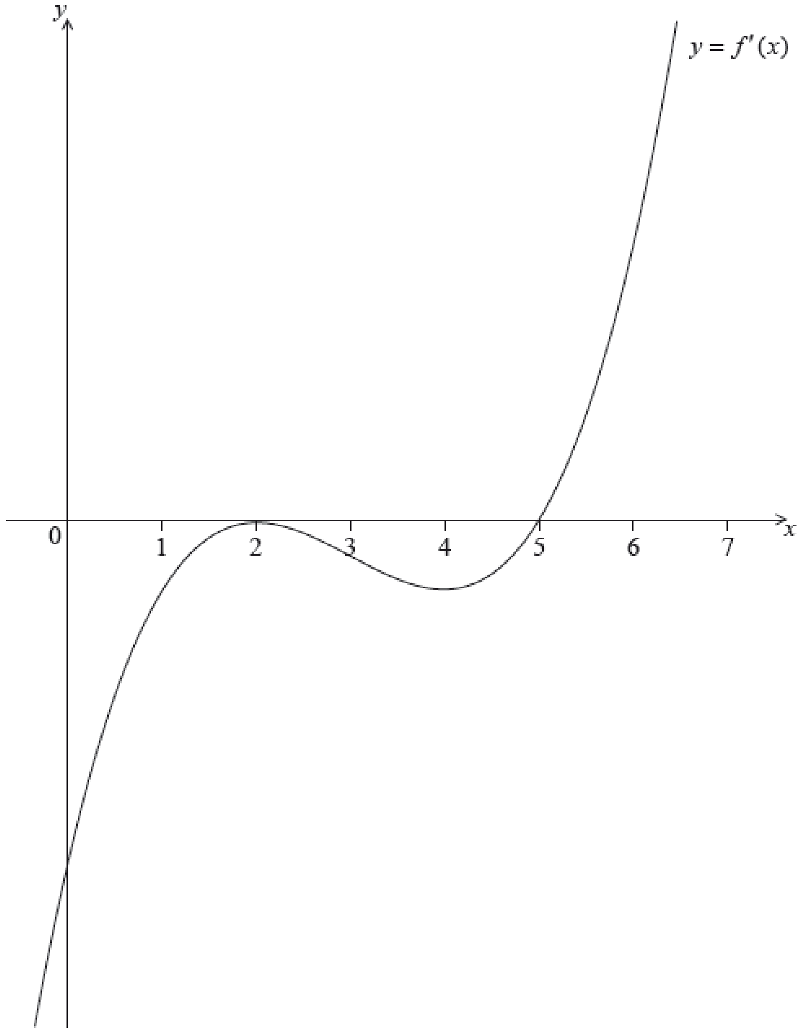


The graph has a local maximum at A, where $x = -2$, and a local minimum at B, where $x = 6$.

a. On the following axes, sketch the graph of $y = f'(x)$. [4]

b. Write down the following in order from least to greatest: $f(0),\ f'(6),\ f''(-2)$. [2]

Let $y = f(x)$, for $-0.5 \le x \le 6.5$. The following diagram shows the graph of f', the derivative of f.

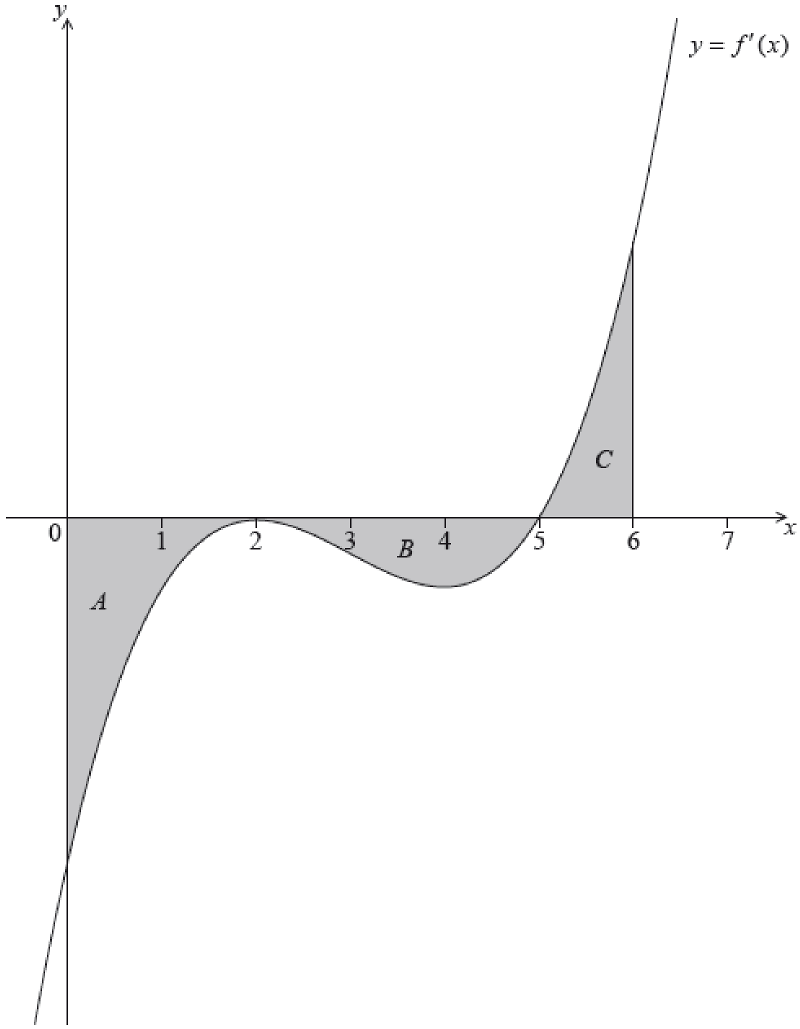


The graph of f' has a local maximum when $x = 2$, a local minimum when $x = 4$, and it crosses the x-axis at the point $(5,\ 0)$.

a. Explain why the graph of f has a local minimum when $x = 5$. [2]

b. Find the set of values of x for which the graph of f is concave down. [2]

c. The following diagram shows the shaded regions A, B and C. [5]

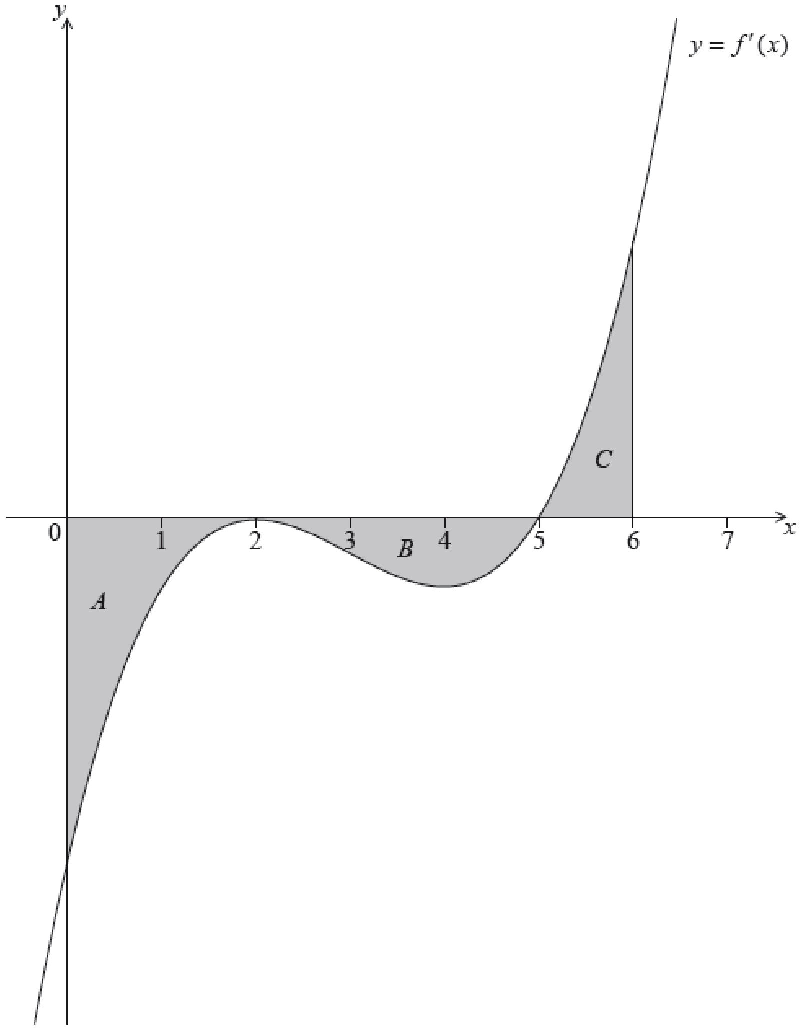


The regions are enclosed by the graph of f', the x-axis, the y-axis, and the line $x = 6$.

The area of region A is 12, the area of region B is 6.75 and the area of region C is 6.75.

Given that $f(0) = 14$, find $f(6)$.

d. The following diagram shows the shaded regions A, B and C. [6]



The regions are enclosed by the graph of f', the *x*-axis, the *y*-axis, and the line $x = 6$.

The area of region A is 12, the area of region B is 6.75 and the area of region C is 6.75.

Let $g(x) = (f(x))^2$. Given that $f'(6) = 16$, find the equation of the tangent to the graph of g at the point where $x = 6$.

Let $\int_1^5 3f(x)\mathrm{d}x = 12$.

a. Show that $\int_5^1 f(x)\mathrm{d}x = -4$. [2]

b. Find the value of $\int_1^2 (x + f(x))\mathrm{d}x + \int_2^5 (x + f(x))\mathrm{d}x$. [5]

A function f has its derivative given by $f'(x) = 3x^2 - 2kx - 9$, where k is a constant.

a. Find $f''(x)$. [2]

b. The graph of f has a point of inflexion when $x = 1$. [3]

Show that $k = 3$.

c. Find $f'(-2)$. [2]

d. Find the equation of the tangent to the curve of f at $(-2,\ 1)$, giving your answer in the form $y = ax + b$. [4]

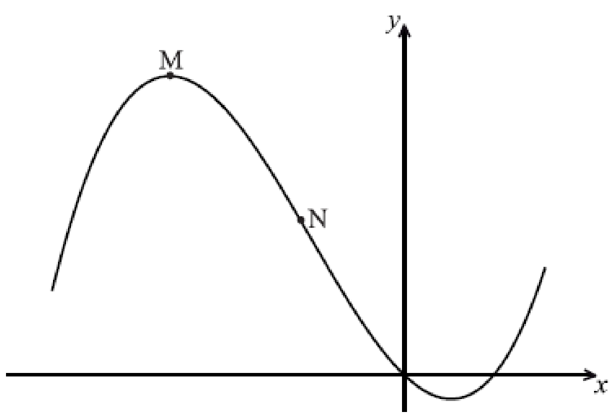
e. Given that $f'(-1) = 0$, explain why the graph of f has a local maximum when $x = -1$. [3]

Consider $f(x) = x^2 \sin x$.

a. Find $f'(x)$. [4]

b. Find the gradient of the curve of f at $x = \frac{\pi}{2}$. [3]

Consider $f(x) = \frac{1}{3}x^3 + 2x^2 - 5x$. Part of the graph of f is shown below. There is a maximum point at M, and a point of inflexion at N.



a. Find $f'(x)$. [3]

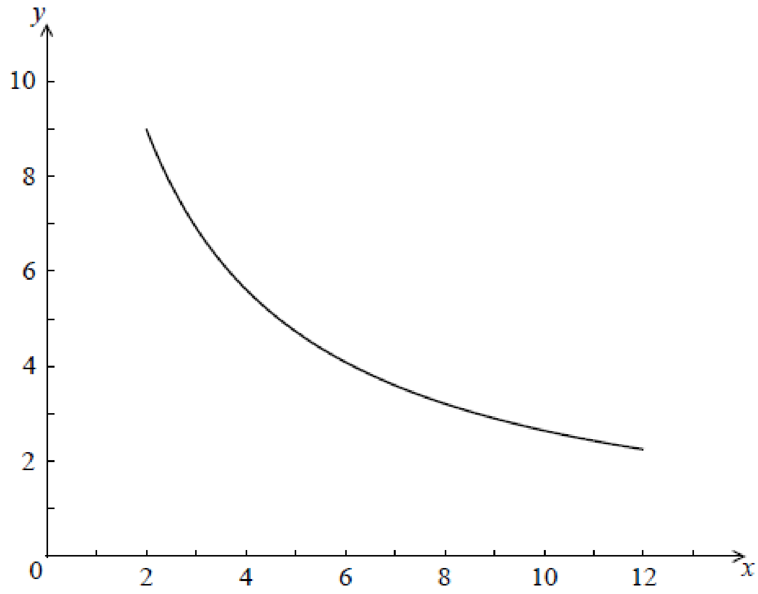
b. Find the x-coordinate of M. [4]

c. Find the x-coordinate of N. [3]

d. The line L is the tangent to the curve of f at $(3, 12)$. Find the equation of L in the form $y = ax + b$. [4]

Let $f(x) = \frac{1}{4}x^2 + 2$. The line L is the tangent to the curve of f at (4, 6) .

Let $g(x) = \frac{90}{3x+4}$, for $2 \leq x \leq 12$. The following diagram shows the graph of g .



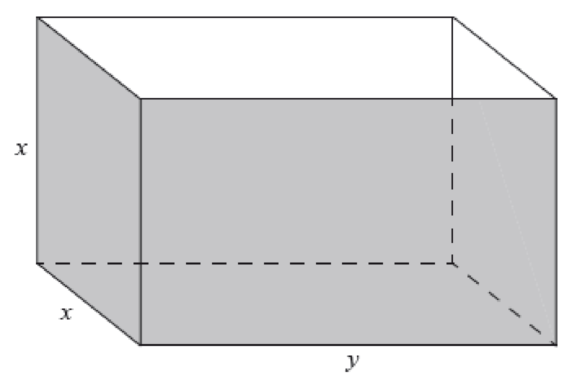
a. Find the equation of L . [4]

b. Find the area of the region enclosed by the curve of g , the x-axis, and the lines $x = 2$ and $x = 12$. Give your answer in the form $a \ln b$, where $a, b \in \mathbb{Z}$. [6]

c. The graph of g is reflected in the x-axis to give the graph of h . The area of the region enclosed by the lines L , $x = 2$, $x = 12$ and the x-axis is 120 120 cm^2 . [3]

Find the area enclosed by the lines L , $x = 2$, $x = 12$ and the graph of h .

Fred makes an open metal container in the shape of a cuboid, as shown in the following diagram.



The container has height x m, width x m and length y m. The volume is 36 m^3.

Let $A(x)$ be the outside surface area of the container.

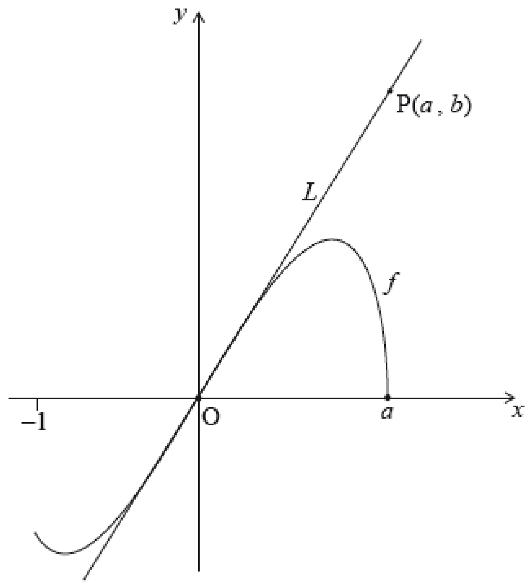
a. Show that $A(x) = \frac{108}{x} + 2x^2$. [4]

b. Find $A'(x)$. [2]

c. Given that the outside surface area is a minimum, find the height of the container. [5]

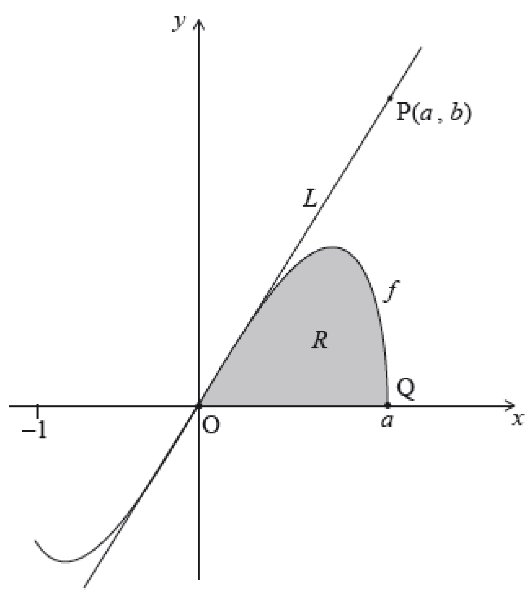
d. Fred paints the outside of the container. A tin of paint covers a surface area of $10\ \mathrm{m}^2$ and costs \$20. Find the total cost of the tins needed to paint the container. [5]

The following diagram shows the graph of $f(x) = 2x\sqrt{a^2 - x^2}$, for $-1 \leqslant x \leqslant a$, where $a > 1$.



The line L is the tangent to the graph of f at the origin, O. The point $\mathrm{P}(a,\ b)$ lies on L.

The point $\mathrm{Q}(a,\ 0)$ lies on the graph of f. Let R be the region enclosed by the graph of f and the x-axis. This information is shown in the following diagram.



Let A_R be the area of the region R.

a. (i) Given that $f'(x) = \frac{2a^2 - 4x^2}{\sqrt{a^2 - x^2}}$, for $-1 \leqslant x < a$, find the equation of L. [6]

(ii) Hence or otherwise, find an expression for b in terms of a.

b. Show that $A_R = \frac{2}{3}a^3$. [6]

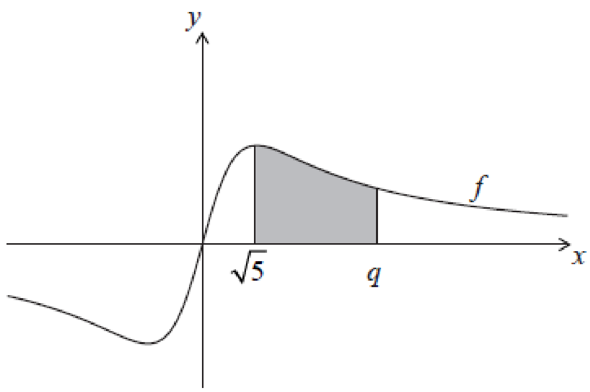
c. Let A_T be the area of the triangle OPQ. Given that $A_T = kA_R$, find the value of k. [4]

Let $f(x) = \frac{2x}{x^2+5}$.

a. Use the quotient rule to show that $f'(x) = \frac{10-2x^2}{(x^2+5)^2}$. [4]

b. Find $\int \frac{2x}{x^2+5} \mathrm{d}x$. [4]

c. The following diagram shows part of the graph of f. [7]



The shaded region is enclosed by the graph of f, the x-axis, and the lines $x = \sqrt{5}$ and $x = q$. This region has an area of $\ln 7$. Find the value of q.

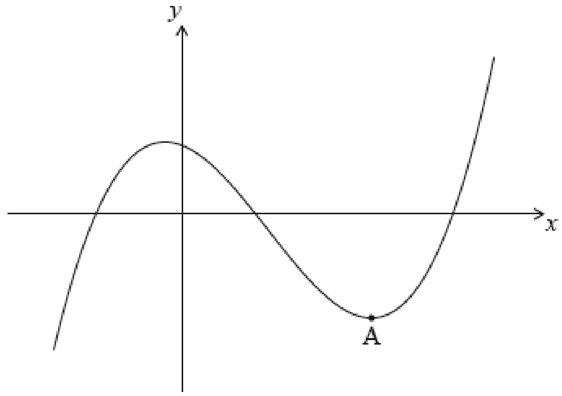
Let $\int_{\pi}^{a} \cos 2x \mathrm{d}x = \frac{1}{2}$, where $\pi < a < 2\pi$. Find the value of a.

A rocket moving in a straight line has velocity v km s^{-1} and displacement s km at time t seconds. The velocity v is given by $v(t) = 6\mathrm{e}^{2t} + t$.

When $t = 0$, $s = 10$.

Find an expression for the displacement of the rocket in terms of t .

The following diagram shows the graph of a function f. There is a local minimum point at A, where $x > 0$.



The derivative of f is given by $f'(x) = 3x^2 - 8x - 3$.

a. Find the x-coordinate of A. [5]

b. The y-intercept of the graph is at $(0, 6)$. Find an expression for $f(x)$. [6]

The graph of a function g is obtained by reflecting the graph of f in the y-axis, followed by a translation of $\begin{pmatrix} m \\ n \end{pmatrix}$.

c. Find the x-coordinate of the local minimum point on the graph of g. [3]

Let $f(x) = 1 + \mathrm{e}^{-x}$ and $g(x) = 2x + b$, for $x \in \mathbb{R}$, where b is a constant.

a. Find $(g \circ f)(x)$. [2]

b. Given that $\lim\limits_{x \to +\infty} (g \circ f)(x) = -3$, find the value of b. [4]

Let $f'(x) = \frac{3x^2}{(x^3+1)^5}$. Given that $f(0) = 1$, find $f(x)$.

A function f has its first derivative given by $f'(x) = (x - 3)^3$.

a. Find the second derivative. [2]

b. Find $f'(3)$ and $f''(3)$. [1]

c. The point P on the graph of f has x-coordinate 3. Explain why P is not a point of inflexion. [2]

Let $f(x) = \frac{6x}{x+1}$, for $x > 0$.

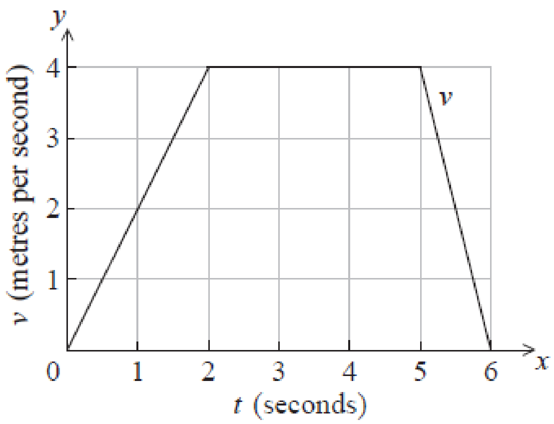
a. Find $f'(x)$. [5]

b. Let $g(x) = \ln\left(\frac{6x}{x+1}\right)$, for $x > 0$. [4]

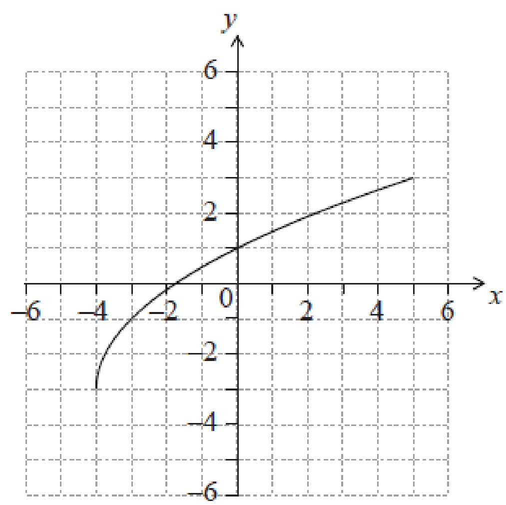
Show that $g'(x) = \frac{1}{x(x+1)}$.

c. Let $h(x) = \frac{1}{x(x+1)}$. The area enclosed by the graph of h , the x-axis and the lines $x = \frac{1}{5}$ and $x = k$ is $\ln 4$. Given that $k > \frac{1}{5}$, find the value of k . [7]

A toy car travels with velocity v ms^{-1} for six seconds. This is shown in the graph below.



The following diagram shows the graph of $y = f(x)$, for $-4 \le x \le 5$.



a. Write down the car's velocity at $t = 3$. [1]

a(i).Write down the value of $f(-3)$; [1]

b. Find the car's acceleration at $t = 1.5$. [2]

c. Find the total distance travelled. [3]

Let L_x be a family of lines with equation given by $r = \begin{pmatrix} x \\ \frac{2}{x} \end{pmatrix} + t\begin{pmatrix} x^2 \\ -2 \end{pmatrix}$, where $x > 0$.

a. Write down the equation of L_1. [2]

b. A line L_a crosses the y-axis at a point P. [6]

Show that P has coordinates $\left(0,\ \frac{4}{a}\right)$.

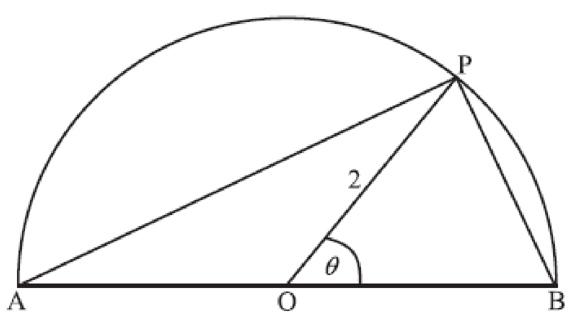
c. The line L_a crosses the x-axis at $\mathrm{Q}(2a,\ 0)$. Let $d = \mathrm{PQ}^2$. [2]

Show that $d = 4a^2 + \frac{16}{a^2}$.

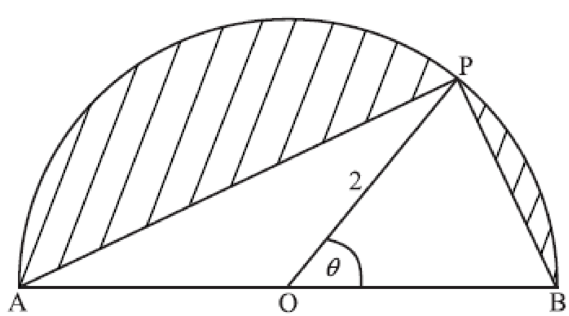
d. There is a minimum value for d. Find the value of a that gives this minimum value. [7]

The following diagram shows a semicircle centre O, diameter [AB], with radius 2.

Let P be a point on the circumference, with $\mathrm{P\hat{O}B} = \theta$ radians.



Let S be the total area of the two segments shaded in the diagram below.



a. Find the area of the triangle OPB, in terms of θ . [2]

b. Explain why the area of triangle OPA is the same as the area triangle OPB. [3]

c. Show that $S = 2(\pi - 2\sin\theta)$. [3]

d. Find the value of θ when S is a local minimum, justifying that it is a minimum. [8]

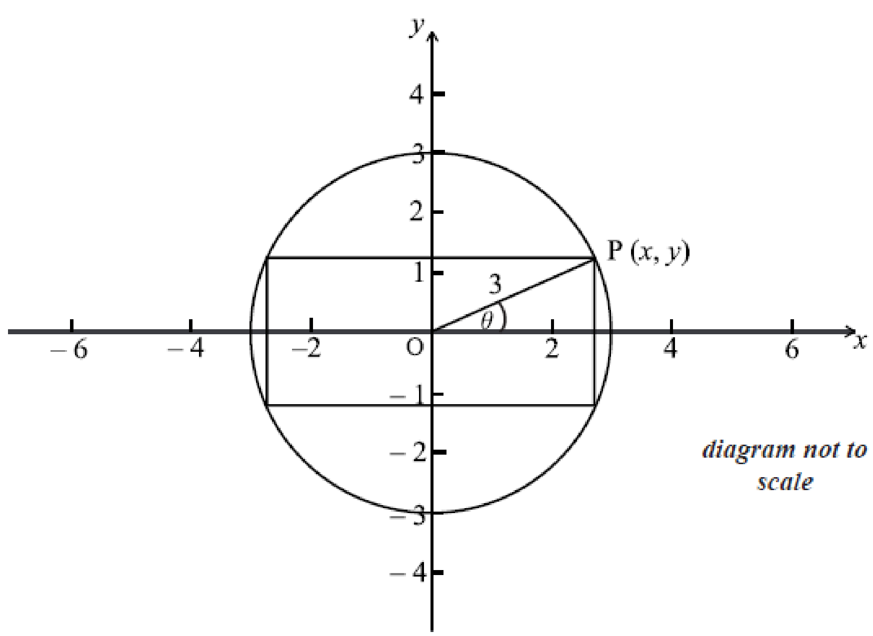
e. Find a value of θ for which S has its greatest value. [2]

Consider a function $f(x)$ such that $\int_1^6 f(x)\mathrm{d}x = 8$.

a. Find $\int_1^6 2f(x)\mathrm{d}x$. [2]

b. Find $\int_1^6 (f(x)+2)\,\mathrm{d}x$. [4]

A rectangle is inscribed in a circle of radius 3 cm and centre O, as shown below.



The point P(x , y) is a vertex of the rectangle and also lies on the circle. The angle between (OP) and the x-axis is θ radians, where $0 \le \theta \le \frac{\pi}{2}$.

a. Write down an expression in terms of θ for [2]

(i) x ;

(ii) y .

b. Let the area of the rectangle be A. [3]

Show that $A = 18\sin 2\theta$.

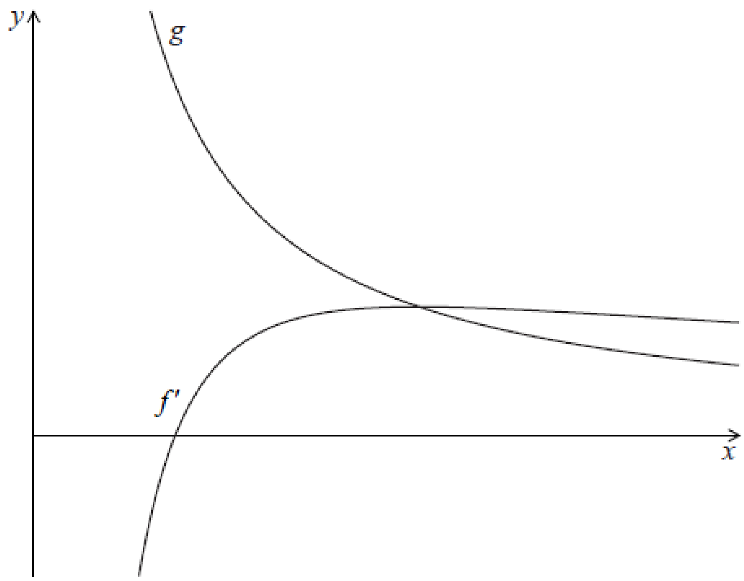
c. (i) Find $\frac{\mathrm{d}A}{\mathrm{d}\theta}$. [8]

(ii) Hence, find the exact value of θ which maximizes the area of the rectangle.

(iii) Use the second derivative to justify that this value of θ does give a maximum.

Let $f(x) = \frac{(\ln x)^2}{2}$, for $x > 0$.

Let $g(x) = \frac{1}{x}$. The following diagram shows parts of the graphs of f' and g.



The graph of f' has an x-intercept at $x = p$.

a. Show that $f'(x) = \frac{\ln x}{x}$. [2]

b. There is a minimum on the graph of f. Find the x-coordinate of this minimum. [3]

c. Write down the value of p. [2]

d. The graph of g intersects the graph of f' when $x = q$. [3]

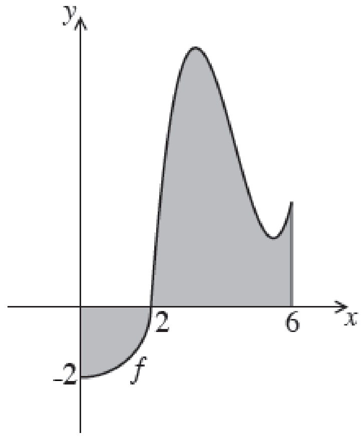
Find the value of q.

e. The graph of g intersects the graph of f' when $x = q$. [5]

Let R be the region enclosed by the graph of f', the graph of g and the line $x = p$.

Show that the area of R is $\frac{1}{2}$.

The following is the graph of a function f , for $0 \leq x \leq 6$.



The first part of the graph is a quarter circle of radius 2 with centre at the origin.

. (a) Find $\int_0^2 f(x)\mathrm{d}x$. [7]

(b) The shaded region is enclosed by the graph of f , the x-axis, the y-axis and the line $x = 6$. The area of this region is 3π .

Find $\int_2^6 f(x)\mathrm{d}x$.

a. Find $\int_0^2 f(x)\mathrm{d}x$. [4]

b. The shaded region is enclosed by the graph of f , the x-axis, the y-axis and the line $x = 6$. The area of this region is 3π . [3]

Find $\int_2^6 f(x)\mathrm{d}x$.

Consider $f(x) = \ln(x^4 + 1)$.

The second derivative is given by $f''(x) = \frac{4x^2(3-x^4)}{(x^4+1)^2}$.

The equation $f''(x) = 0$ has only three solutions, when $x = 0$, $\pm\sqrt[4]{3}$ $(\pm 1.316\ldots)$.

a. Find the value of $f(0)$. [2]

b. Find the set of values of x for which f is increasing. [5]

c. (i) Find $f''(1)$. [5]

(ii) **Hence**, show that there is no point of inflexion on the graph of f at $x = 0$.

d. There is a point of inflexion on the graph of f at $x = \sqrt[4]{3}$ $(x = 1.316\ldots)$. [3]

Sketch the graph of f , for $x \geq 0$.

The acceleration, a ms^{-2}, of a particle at time t seconds is given by $a = 2t + \cos t$.

a. Find the acceleration of the particle at $t = 0$. [2]

b. Find the velocity, v, at time t, given that the initial velocity of the particle is ms^{-1} . [5]

c. Find $\int_0^3 v\mathrm{d}t$, giving your answer in the form $p - q\cos 3$. [7]

d. What information does the answer to part (c) give about the motion of the particle? [2]

Let $f : x \mapsto \sin^3 x$.

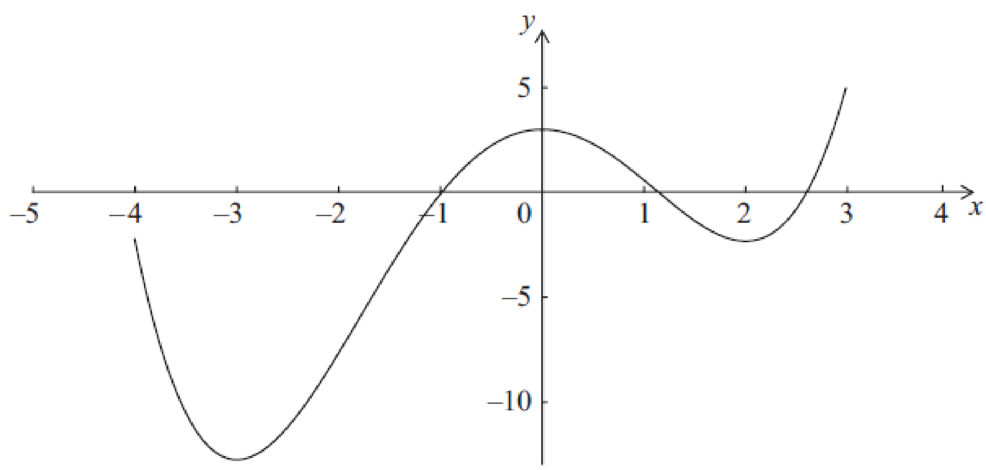
a. (i) Write down the range of the function f . [5]

(ii) Consider $f(x) = 1$, $0 \le x \le 2\pi$. Write down the number of solutions to this equation. Justify your answer.

b. Find $f'(x)$, giving your answer in the form $a\sin^p x\cos^q x$ where a, p, $q \in \mathbb{Z}$. [2]

c. Let $g(x) = \sqrt{3}\sin x(\cos x)^{\frac{1}{2}}$ for $0 \le x \le \frac{\pi}{2}$. Find the volume generated when the curve of g is revolved through 2π about the x-axis. [7]

A function f is defined for $-4 \le x \le 3$. The graph of f is given below.



The graph has a local maximum when $x = 0$, and local minima when $x = -3$, $x = 2$.

a. Write down the x-intercepts of the graph of the **derivative** function, f' . [2]

b. Write down all values of x for which $f'(x)$ is positive. [2]

c. At point D on the graph of f, the x-coordinate is -0.5. Explain why $f''(x) < 0$ at D. [2]

The graph of a function h passes through the point $\left(\frac{\pi}{12}, 5\right)$.

Given that $h'(x) = 4\cos 2x$, find $h(x)$.

Let $f(x) = \mathrm{e}^{-3x}$ and $g(x) = \sin\left(x - \frac{\pi}{3}\right)$.

a. Write down [2]

(i) $f'(x)$;

(ii) $g'(x)$.

b. Let $h(x) = \mathrm{e}^{-3x}\sin\left(x - \frac{\pi}{3}\right)$. Find the exact value of $h'\left(\frac{\pi}{3}\right)$. [4]

The velocity v ms^{-1} of a particle at time t seconds, is given by $v = 2t + \cos 2t$, for $0 \le t \le 2$.

a. Write down the velocity of the particle when $t = 0$. [1]

b(i) and (ii). When $t = k$, the acceleration is zero. [8]

(i) Show that $k = \frac{\pi}{4}$.

(ii) Find the exact velocity when $t = \frac{\pi}{4}$.

c. When $t < \frac{\pi}{4}$, $\frac{\mathrm{d}v}{\mathrm{d}t} > 0$ and when $t > \frac{\pi}{4}$, $\frac{\mathrm{d}v}{\mathrm{d}t} > 0$. [4]

Sketch a graph of v against t .

d(i) and (ii). Let d be the distance travelled by the particle for $0 \le t \le 1$. [3]

(i) Write down an expression for d .

(ii) Represent d on your sketch.

In this question, you are given that $\cos\frac{\pi}{3} = \frac{1}{2}$, and $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

The displacement of an object from a fixed point, O is given by $s(t) = t - \sin 2t$ for $0 \le t \le \pi$.

a. Find $s'(t)$. [3]

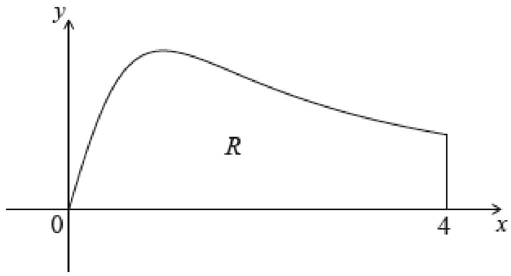
b. In this interval, there are only two values of t for which the object is not moving. One value is $t = \frac{\pi}{6}$. [4]

Find the other value.

c. Show that $s'(t) > 0$ between these two values of t . [3]

d. Find the distance travelled between these two values of t . [5]

The following diagram shows the graph of $f(x) = \frac{x}{x^2+1}$, for $0 \leq x \leq 4$, and the line $x = 4$.

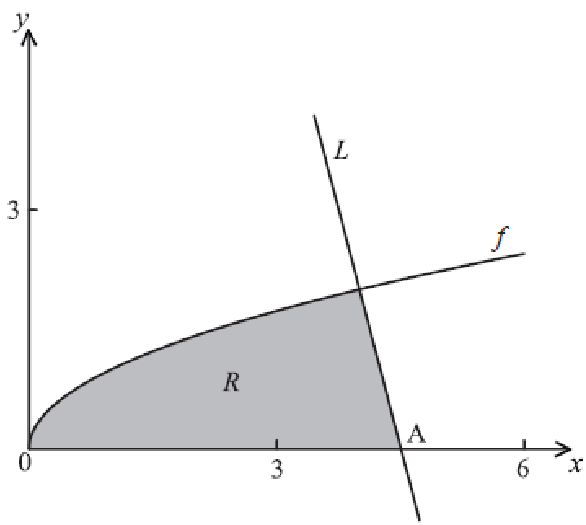


Let R be the region enclosed by the graph of f , the x-axis and the line $x = 4$.

Find the area of R.

Let $f(x) = \sqrt{x}$. Line L is the normal to the graph of f at the point (4, 2) .

In the diagram below, the shaded region R is bounded by the x-axis, the graph of f and the line L .



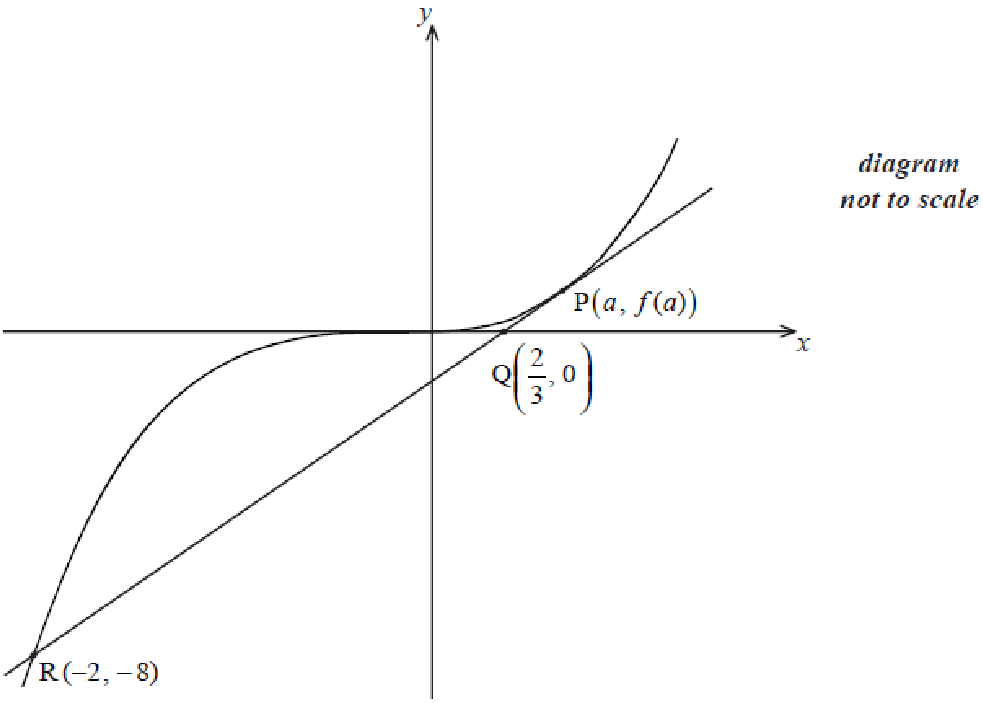
a. Show that the equation of L is $y = -4x + 18$. [4]

b. Point A is the x-intercept of L . Find the x-coordinate of A. [2]

c. Find an expression for the area of R . [3]

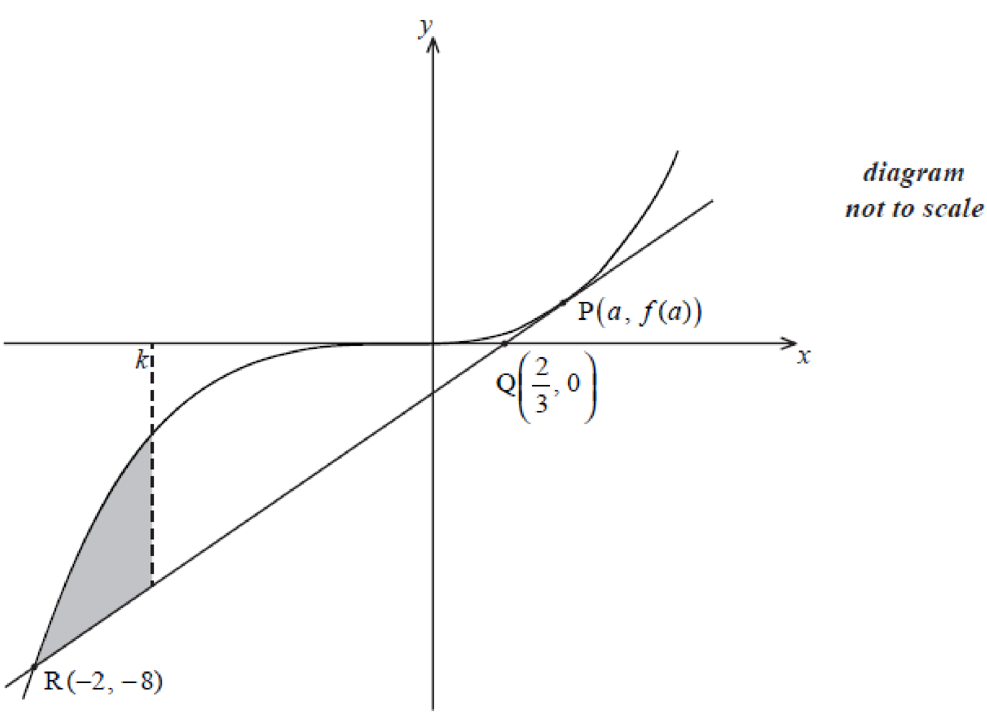
d. The region R is rotated 360° about the x-axis. Find the volume of the solid formed, giving your answer in terms of π . [8]

Let $f(x) = x^3$. The following diagram shows part of the graph of f .



The point $P(a, f(a))$, where $a > 0$, lies on the graph of f. The tangent at P crosses the x-axis at the point $Q\left(\frac{2}{3}, 0\right)$. This tangent intersects the graph of f at the point $R(-2, -8)$.

The equation of the tangent at P is $y = 3x - 2$. Let T be the region enclosed by the graph of f, the tangent [PR] and the line $x = k$, between $x = -2$ and $x = k$ where $-2 < k < 1$. This is shown in the diagram below.



a(i),(ii) and (iii) (i) Show that the gradient of [PQ] is $\frac{a^3}{a-\frac{2}{3}}$. [7]

(ii) Find $f'(a)$.

(iii) Hence show that $a = 1$.

b. Given that the area of T is $2k+4$, show that k satisfies the equation $k^4 - 6k^2 + 8 = 0$. [9]

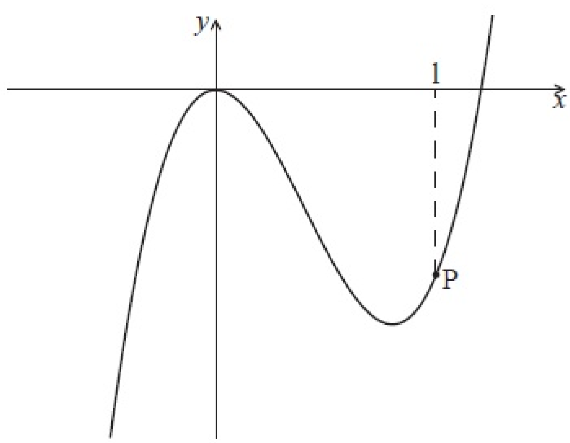
A quadratic function f can be written in the form $f(x) = a(x-p)(x-3)$. The graph of f has axis of symmetry $x = 2.5$ and y-intercept at $(0,\ -6)$

a. Find the value of p. [3]

b. Find the value of a. [3]

c. The line $y = kx - 5$ is a tangent to the curve of f. Find the values of k. [8]

Part of the graph of $f(x) = ax^3 - 6x^2$ is shown below.



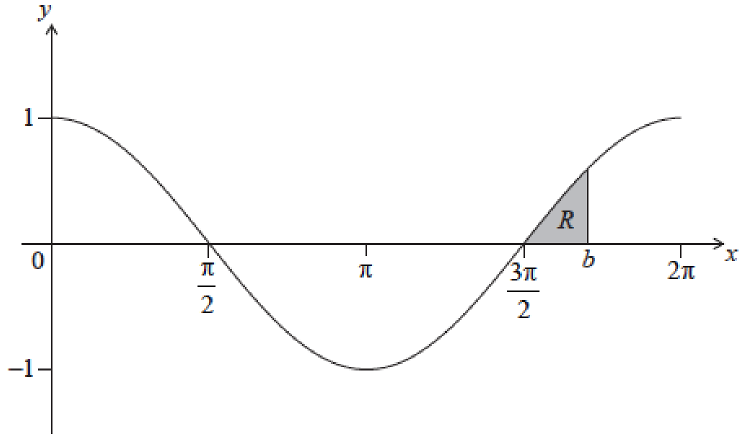
The point P lies on the graph of f . At P, $x = 1$.

a. Find $f'(x)$. [2]

b. The graph of f has a gradient of 3 at the point P. Find the value of a . [4]

Let $f(x) = \cos x$, for $0 \le x \le 2\pi$. The following diagram shows the graph of f.

There are x-intercepts at $x = \frac{\pi}{2},\ \frac{3\pi}{2}$.



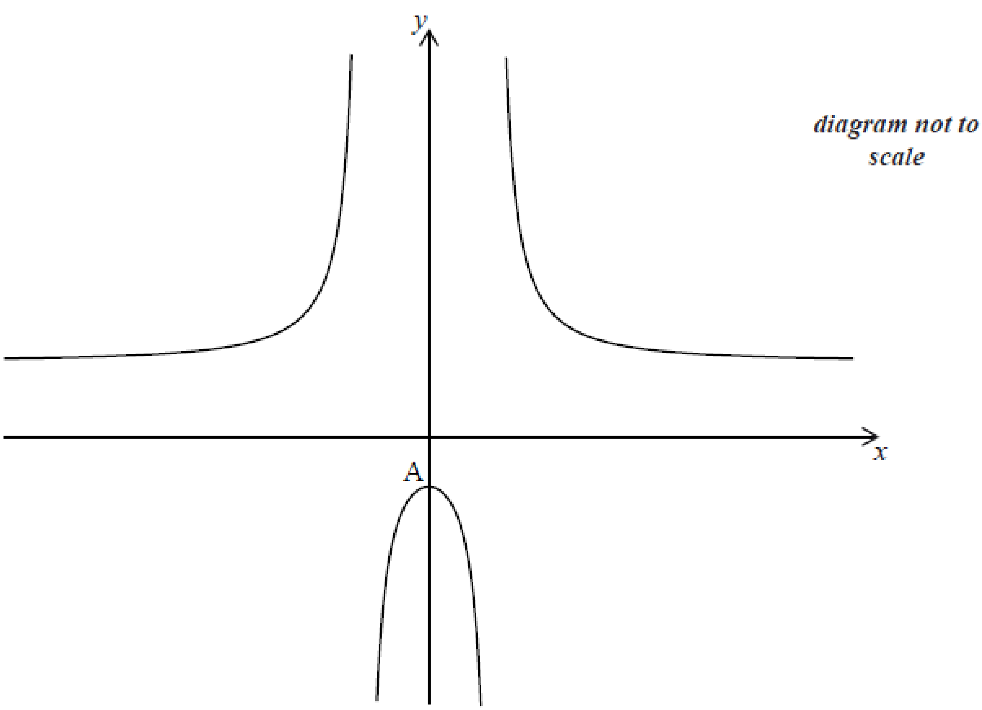
The shaded region R is enclosed by the graph of f, the line $x = b$, where $b > \frac{3\pi}{2}$, and the x-axis. The area of R is $\left(1 - \frac{\sqrt{3}}{2}\right)$. Find the value of b.

Let $f(x) = px^3 + px^2 + qx$.

a. Find $f'(x)$. [2]

b. Given that $f'(x) \geqslant 0$, show that $p^2 \leqslant 3pq$. [5]

Let $f(x) = 3 + \frac{20}{x^2-4}$, for $x \neq \pm 2$. The graph of f is given below.



The y-intercept is at the point A.

a. (i) Find the coordinates of A. [7]

(ii) Show that $f'(x) = 0$ at A.

b. The second derivative $f''(x) = \frac{40(3x^2+4)}{(x^2-4)^3}$. Use this to [6]

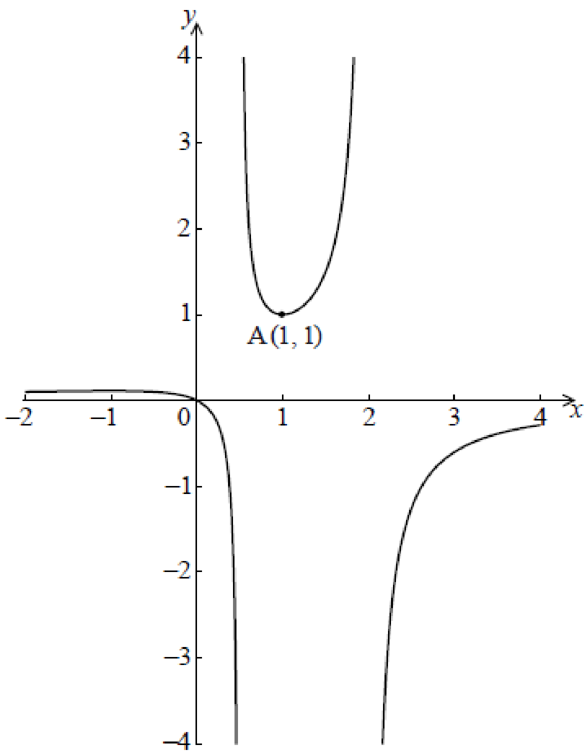
(i) justify that the graph of *f* has a local maximum at A;

(ii) explain why the graph of *f* does **not** have a point of inflexion.

c. Describe the behaviour of the graph of f for large $|x|$. [1]

d. Write down the range of f . [2]

Let $f(x) = \frac{x}{-2x^2+5x-2}$ for $-2 \le x \le 4$, $x \ne \frac{1}{2}$, $x \ne 2$. The graph of f is given below.



The graph of f has a local minimum at A(1, 1) and a local maximum at B.

a. Use the quotient rule to show that $f'(x) = \frac{2x^2-2}{(-2x^2+5x-2)^2}$. [6]

b. Hence find the coordinates of B. [7]

c. Given that the line $y = k$ does not meet the graph of *f* , find the possible values of *k* . [3]

Let $f'(x) = 6x^2 - 5$. Given that $f(2) = -3$, find $f(x)$.

Consider $f(x) = \log k(6x - 3x^2)$, for $0 < x < 2$, where $k > 0$.

The equation $f(x) = 2$ has exactly one solution. Find the value of k.

Let $f'(x) = 3x^2 + 2$. Given that $f(2) = 5$, find $f(x)$.

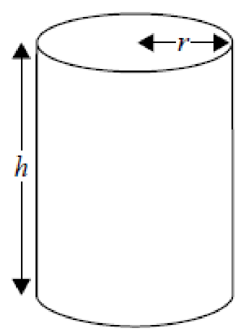
Let $f(x) = \int \frac{12}{2x-5} \mathrm{d}x$, $x > \frac{5}{2}$. The graph of f passes through (4, 0) .

Find $f(x)$.

Let $f'(x) = \sin^3(2x)\cos(2x)$. Find $f(x)$, given that $f\left(\frac{\pi}{4}\right) = 1$.

A closed cylindrical can with radius r centimetres and height h centimetres has a volume of 20π cm^3.

diagram not to scale



The material for the base and top of the can costs 10 cents per cm^2 and the material for the curved side costs 8 cents per cm^2. The total cost of the material, in cents, is *C*.

a. Express *h* in terms of *r*. [2]

b. Show that $C = 20\pi r^2 + \frac{320\pi}{r}$. [4]

c. Given that there is a minimum value for *C*, find this minimum value in terms of π. [9]

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a. Find $\int \frac{e^x}{1+e^x} dx$. [3]

b. Find $\int \sin 3x \cos 3x dx$. [4]

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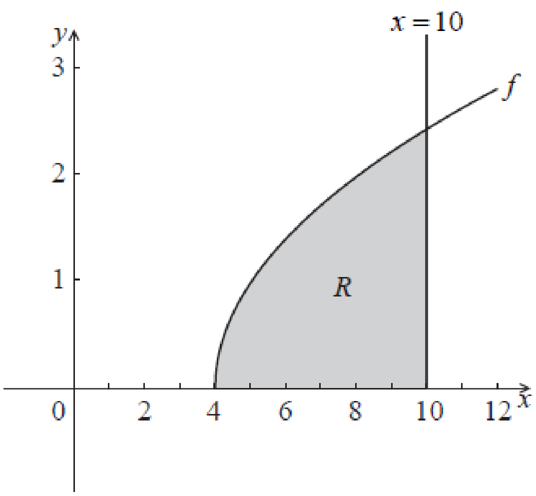
a. Find $\int_4^{10} (x-4) dx$. [4]

b. Part of the graph of $f(x) = \sqrt{x-4}$, for $x \geq 4$, is shown below. The shaded region R is enclosed by the graph of f , the line $x = 10$, and the x-axis. [3]



The region R is rotated 360° about the x-axis. Find the volume of the solid formed.

A particle moves along a straight line so that its velocity, v ms^{-1} at time t seconds is given by $v = 6e^{3t} + 4$. When $t = 0$, the displacement, s, of the particle is 7 metres. Find an expression for s in terms of t.

Consider the function f with second derivative $f''(x) = 3x - 1$. The graph of f has a minimum point at A(2, 4) and a maximum point at B$\left(-\frac{4}{3}, \frac{358}{27}\right)$.

a. Use the second derivative to justify that B is a maximum. [3]

b. Given that $f'(x) = \frac{3}{2}x^2 - x + p$, show that $p = -4$. [4]

c. Find $f(x)$. [7]

Let $f'(x) = \frac{6-2x}{6x-x^2}$, for $0 < x < 6$.

The graph of f has a maximum point at P.

The y-coordinate of P is $\ln 27$.

a. Find the x-coordinate of P. [3]

b. Find $f(x)$, expressing your answer as a single logarithm. [8]

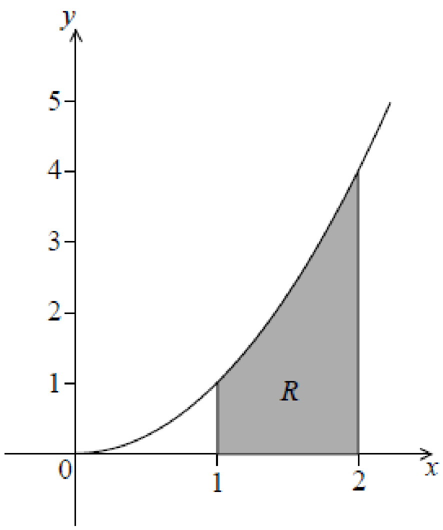
c. The graph of f is transformed by a vertical stretch with scale factor $\frac{1}{\ln 3}$. The image of P under this transformation has coordinates $(a,\ b)$. [[N/A

Find the value of a and of b, where $a,\ b \in \mathbb{N}$.

Let $f(x) = x^2$.

a. Find $\int_1^2 (f(x))^2 \mathrm{d}x$. [4]

b. The following diagram shows part of the graph of f. [2]



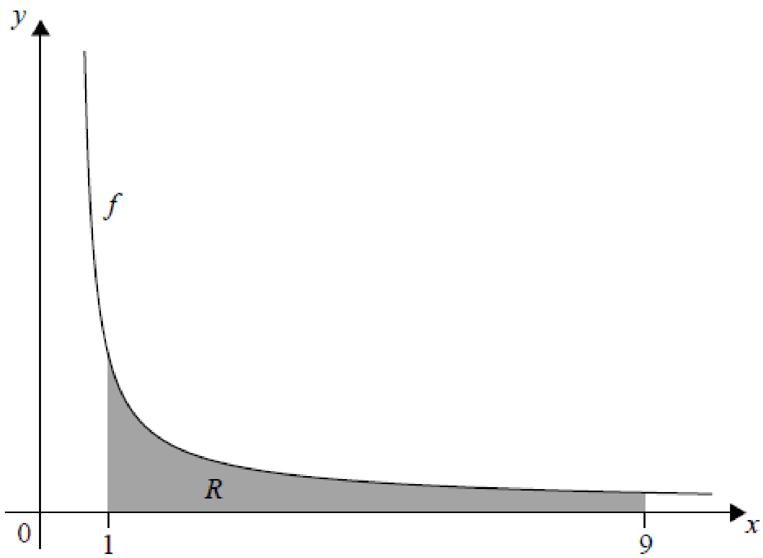
The shaded region R is enclosed by the graph of f, the x-axis and the lines $x = 1$ and $x = 2$.
Find the volume of the solid formed when R is revolved 360° about the x-axis.

Given that $\int_0^5 \frac{2}{2x+5} \mathrm{d}x = \ln k$, find the value of k .

Let $f\,(x) = \frac{1}{\sqrt{2x-1}}$, for $x > \frac{1}{2}$.

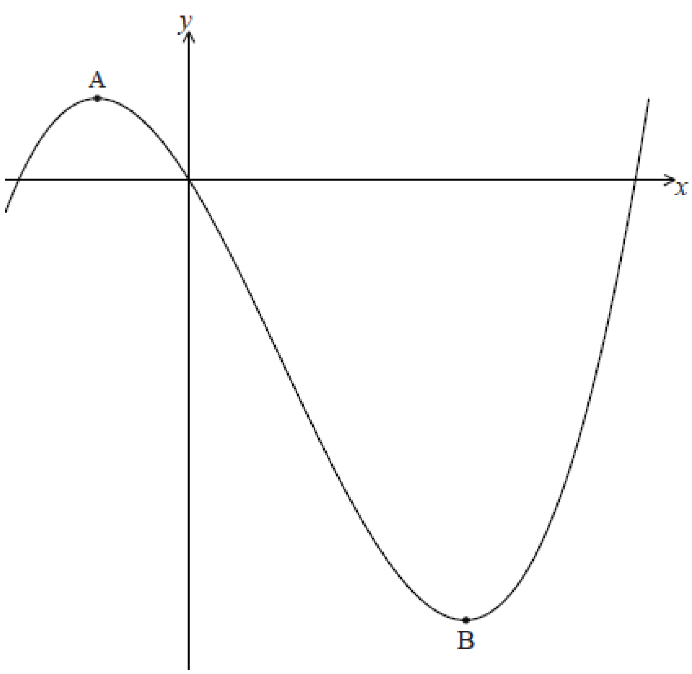
a. Find $\int (f\,(x))^2 \mathrm{d}x$. [3]

b. Part of the graph of *f* is shown in the following diagram. [4]



The shaded region *R* is enclosed by the graph of *f*, the *x*-axis, and the lines *x* = 1 and *x* = 9 . Find the volume of the solid formed when *R* is revolved 360° about the *x*-axis.

Let $f(x) = \frac{1}{2}x^3 - x^2 - 3x$. Part of the graph of *f* is shown below.



There is a maximum point at A and a minimum point at B(3, − 9) .

a. Find the coordinates of A. [8]

b(i), (ii) and (iii) Write down the coordinates of [6]

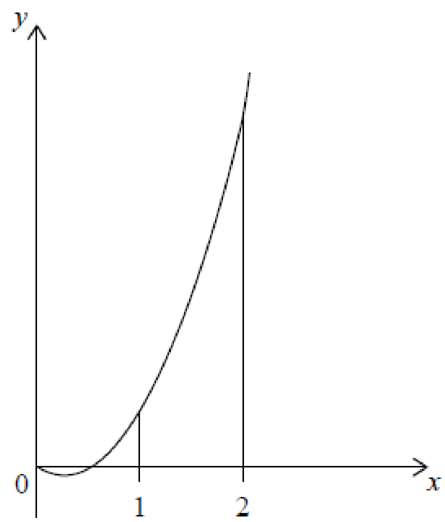
(i) the image of B after reflection in the *y*-axis;

(ii) the image of B after translation by the vector $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$;

(iii) the image of B after reflection in the x-axis followed by a horizontal stretch with scale factor $\frac{1}{2}$.

The graph of $y = \sqrt{x}$ between $x = 0$ and $x = a$ is rotated $360°$ about the x-axis. The volume of the solid formed is 32π . Find the value of a.

Let $f(x) = 6x^2 - 3x$. The graph of f is shown in the following diagram.



a. Find $\int (6x^2 - 3x)\,dx$. [2]

b. Find the area of the region enclosed by the graph of f, the x-axis and the lines $x = 1$ and $x = 2$. [4]

The following table shows the probability distribution of a discrete random variable A, in terms of an angle θ.

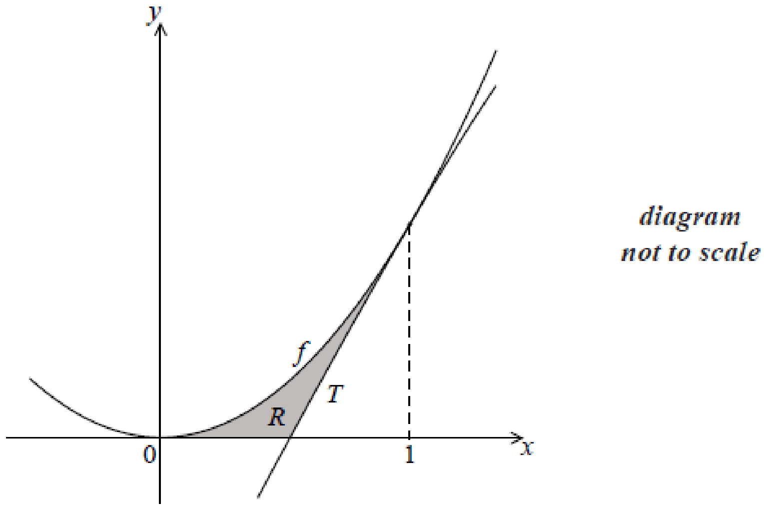
a	1	2
$P(A = a)$	$\cos\theta$	$2\cos 2\theta$

a. Show that $\cos\theta = \frac{3}{4}$. [6]

b. Given that $\tan\theta > 0$, find $\tan\theta$. [3]

c. Let $y = \frac{1}{\cos x}$, for $0 < x < \frac{\pi}{2}$. The graph of $y$between $x = \theta$ and $x = \frac{\pi}{4}$ is rotated 360° about the x-axis. Find the volume of the solid formed. [6]

The following diagram shows part of the graph of the function $f(x) = 2x^2$.



The line T is the tangent to the graph of f at $x = 1$.

a. Show that the equation of T is $y = 4x - 2$. [5]

b. Find the x-intercept of T . [2]

c(i) and (ii). The shaded region R is enclosed by the graph of f , the line T , and the x-axis. [9]

(i) Write down an expression for the area of R .

(ii) Find the area of R .

Let $g(x) = \frac{\ln x}{x^2}$, for $x > 0$.

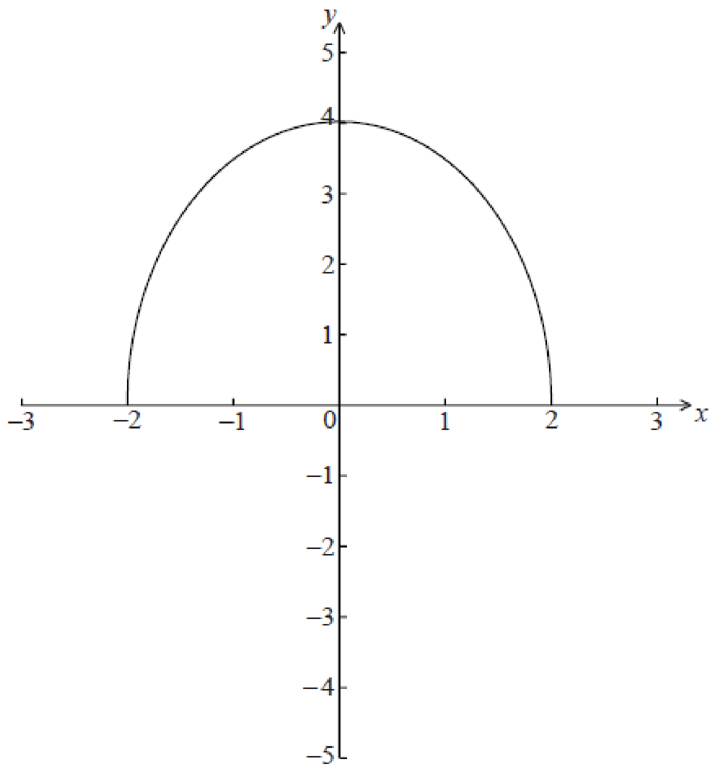
a. Use the quotient rule to show that $g'(x) = \frac{1 - 2\ln x}{x^3}$. [4]

b. The graph of g has a maximum point at A. Find the x-coordinate of A. [3]

Consider $f(x)$, $g(x)$ and $h(x)$, for $x \in \mathbb{R}$ where $h(x) = (f \circ g)(x)$.

Given that $g(3) = 7$, $g'(3) = 4$ and $f'(7) = -5$, find the gradient of the normal to the curve of h at $x = 3$.

The graph of $f(x) = \sqrt{16 - 4x^2}$, for $-2 \leq x \leq 2$, is shown below.



The region enclosed by the curve of *f* and the *x*-axis is rotated 360° about the *x*-axis.

Find the volume of the solid formed.

Let $f(x) = \mathrm{e}^{2x}$. The line L is the tangent to the curve of f at $(1,\ \mathrm{e}^2)$.
Find the equation of L in the form $y = ax + b$.

Let $f(x) = \mathrm{e}^x \cos x$. Find the gradient of the normal to the curve of *f* at $x = \pi$.

Let $h(x) = \frac{6x}{\cos x}$. Find $h'(0)$.

A function $f(x)$ has derivative $f'(x) = 3x^2 + 18x$. The graph of *f* has an *x*-intercept at $x = -1$.

a. Find $f(x)$. [6]

b. The graph of *f* has a point of inflexion at $x = p$. Find p. [4]

c. Find the values of *x* for which the graph of *f* is concave-down. [3]

Let $f(x) = \sin x + \frac{1}{2}x^2 - 2x$, for $0 \leq x \leq \pi$.

Let g be a quadratic function such that $g(0) = 5$. The line $x = 2$ is the axis of symmetry of the graph of g .

The function g can be expressed in the form $g(x) = a(x - h)^2 + 3$.

a. Find $f'(x)$. [3]

b. Find $g(4)$. [3]

c. (i) Write down the value of h . [4]

(ii) Find the value of a .

d. Find the value of x for which the tangent to the graph of f is parallel to the tangent to the graph of g . [6]
